

Approximation-by-conjugation method (AbC) was introduced in the 70-th by D.V. Anosov and A.B. Katok. By now it has become one of the main tools in constructing examples of smooth dynamics with prescribed chaotic properties. In this course we will present the general method and discuss several useful ideas for its application. Here is a more detailed plan and a list of papers that will be referred to during the lectures.

Lecture 1. The general idea of the method. Examples of exotic analytic behavior on the torus.

References: [AK], [S], [FS05], [FH].

Lecture 2. Smooth chaotic dynamics via rearrangement of partitions. Moser's trick.

References: [FS05] [AK], [M], [McDS], [Be1] [BKu19].

Lecture 3. Smooth weakly-mixing diffeomorphism on the disk with any prescribed Liouville rotation on the boundary. Other examples.

References: [FS05], [FK04], [KS], [KR].

Lecture 4. AbC constructions in the Hamiltonian context. Isolated invariant torus of arbitrary frequency for a smooth Hamiltonian.

References: [EFK] (see also [H98], [McDS]).

Lecture 5. More exotic examples in the Hamiltonian context: diffusion at all times; coexistence of integrable and diffusive behavior for an analytic Hamiltonian system.

References: [FS19]; [FF]. Other examples: [DFS], [FW22], [BKu22].

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